# Basic Foundations of the Microscopic Theory of Superconductivity

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## Abstract

A new approach based on macro-orbital representation of a conduction electron in a solid has been used to discover some untouched aspects of the phonon induced attraction between two electrons and to lay the basic foundations of a general theory of superconductivity applicable to widely different solids. To this effect we first analyze the net hamiltonian, $H(N)$, of $N$ conduction electrons to identify its universal part, $H_o(N)$ (*-independent of the nature of a specific solid or a specific class of solids*), and then study the states of $H_o(N)$ to conclude that superconductivity originates, basically, from an inter-play between the zero-point force ($f_o$) of conduction electrons in their ground state and the inter-atomic forces ($f_a$) which decide the lattice structure. This renders a kind of mechanical strain in the lattice which serves as the main source of phonon induced inter-electron attraction responsible for the formation of Cooper type pairs and the onset of superconductivity below certain temperature $T_c$. We determine the binding energy of such pairs and find a relation for $T_c$ which not only accounts for the highest experimental $T_c \approx 135$K that we know to-day but also indicates that superconductivity may, in principle, occur at room temperature. It is evident that electrical strain in the lattice (*i.e.* electrical polarization of the lattice constituents produced by the charge of conducting electrons) can have an added contribution to the phonon induced attraction of two electrons. Our theoretical framework not only incorporates BCS model but also provides microscopic basis for the two well known phenomenologies of superconductivity, *viz.*, the two fluid theory and $\Psi-$theory. In addition, it also corroborates a recent idea that superconducting transition is basically a quantum phase transition.

## 1.0 Introduction

The experimental discovery of high $T_c$ (HTC) superconductivity in 1987 [1] came as a great surprise to the physics community, basically, for its challenge to the Bardeen, Cooper and Schrieffer (BCS) model [2] which had emerged as a highly successful theory of superconductors that we knew at that time. Consequently, HTC systems became a subject of intense research activity and thousands of experimental and theoretical papers have been published over the last eighteen years. While the important results of various

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experimental studies on HTC systems are reviewed in [3-13], the status of our present theoretical understanding is elegantly summed up in [3, 12-20]; references to other reviews and important research articles can be obtained from [3-21].

Several theoretical models based on widely different exotic ideas have been worked out, since no single mechanism could be identified as the basic origin of different properties of HTC systems. We have theories based on Hubbard model [17, 22-24] , spin bag theories [25, 26], antiferromagnetic Fermi-liquid theory [27], $d_{x^2-y^2}$ theories [28, 29], anyon theory [30], bipolaron theory [31] and theories based on the proximity effects of quantum phase transition [15] and it may be mentioned that this list is not exhaustive; the references to many other models can be traced from [3, 12-21].

It is evident that, even after a period of nearly two decades of the discovery of HTC systems, the goal of having a single microscopic theory of superconductivity is far from being achieved. Incidentally, the process of achieving this goal has been frustrated further by certain experimental results, *viz.* : (i) the coexistence of superconductivity with ferro-magnetism [32], (ii) superconductivity of $MgB_2$ at $T_c(\approx 39)$K [33], (iii) pressure/strain induced superconductivity [34], (iv) stripes of charges in a HTC system [35], enhancement of superconductivity by nano-engineered magnetic field in the form of tiny magnetic dots [36], *etc.* as well as by interesting theoretical models which consider two energy gaps [37], formation of Cooper type pairs through spin-spin interaction [38], triplet p-wave pairing and singlet d-wave pairing [39], *etc.* for specific superconductors. As such we either have a system specific theory or a class (*i.e.* a set of superconducting solids) specific theory of superconductivity and numerous ideas that have greatly muddled the selection of right idea(s) which may help in developing a unified single microscopic theory (*preferably incorporating BCS theory*) of the phenomenon. However, we found a way-out by using a non-conventional approach to the problem based on the macro-orbital representation of a quantum particle [40]. Our approach makes no assumption about the nature of the order parameter of superconducting transition or the nature and strength of the interaction responsible for the formation of Cooper type pairs. It simply banks upon the solutions of the Schrödinger equation of $N$ conduction electrons.

We note that conduction electrons form a kind of Fermi fluid which flows through the lattice structure of a solid. To a good approximation, each electron can be identified as a freely moving particle which can be represented by a plane wave unless it suffers collisions with other electron(s) or lattice. It is argued that electrostatic screening effect of the lattice significantly reduces the strength and range of electron-electron repulsion and thereby facilitates the formation of Cooper pairs basically responsible for the phenomenon [2]. However, these effects in certain superconductors (*e.g.* in HTC systems) are found to be relatively weak and each theoretical model of such superconductors looks for a possible source of relatively stronger attraction so that the formation of Cooper type pairs of charge carriers becomes possible. But we make no such argument and discover the real interaction from our theoretical analysis.

In a recent conference [40], we presented our approach of macro-orbital representation of a conduction electron to lay the foundations of a viable theory of superconductivity. We discovered different aspects of this representation, for the first time, in our recent study

of the wave mechanics of two *hard core* (HC) identical particles in 1-D box [41] and used them to understand the unification of the physics of widely different many body systems of interacting bosons and fermions [42] and reveal the ground state of $N$ HC quantum particles in 1-D box [43]. While our approach also concludes Cooper type pairs as the origin of superconductivity but it discovers some untouched aspects of the electron-phonon interaction responsible for the formation of such pairs.

The paper has been arranged as follows. The Hamiltonian of the electron fluid in a solid has been analyzed in Section 2.0 to identify its universal component ($H_o(N)$, Eqn. 2, below), -independent of the specific aspects of a superconductor or a class of superconductors, while the wave mechanics of a pair of conduction electrons, found to serve as the basic unit of the fluid, has been examined in Section 3.0 to conclude its several important aspects and to discover that a conduction electron is better represented by a macro-orbital (*a kind of pair waveform* as described in Section 3.4.7) rather than a plane wave. A wave function that represents a general state of the electron fluid has been constructed in Section 4.0 by using $N$ macro-orbitals for $N$ conduction electrons and used to conclude their ground state configuration. While the equation of state of the electron fluid has been analyzed to obtain the free energy in Section 5.0, different aspects related to superconductivity, such as criticality of electron fluid, onset of lattice strain, energy gap and formation of ($\mathbf{q}$, -$\mathbf{q}$) bound pairs, transition temperature, *etc.* are discussed in Section 6.0. The paper is summed up by examining the consistency of our model with other well known models such as BCS theory, two fluid theory, $\Psi-$theory, *etc.* in Section 7.0 and making certain important remarks in Section 8.0.

For the first time, this paper identifies mechanical strain in the lattice (produced by the zero-point force of the conducting electrons) as *the main factor* responsible for electron-phonon interaction leading to superconductivity; this strain is different from the electrical strain (*i.e.* electrical polarization of the lattice constituents produced by electron charge) emphasized in BCS theory. We believe that electrical strain possibly adds to above mentioned electron-phonon interaction. Since the *zero-point force* is a consequence, purely, of the wave nature of a quantum particle like electron, the onset of the said mechanical strain below certain temperature (*cf.* Section 6.2) rightly represents a universal aspect of superconductivity. It is interesting to note that recent experimental studies confirm the occurrence of lattice strain [44] and corroborate the fact that phonons have major role in the mechanism of superconductivity even in HTC systems [45]. We note that several theoretical studies [46] relate charge fluctuation, spin fluctuation, phase fluctuation, superconducting density fluctuation and/or similar other factors with the onset of superconductivity, while the present analysis sees a possibility of their coupling with the mechanical strain in the lattice which serves as the basic order parameter of the transition.

## 2.0 Important Aspects of The Electron Fluid

### 2.1. Hamiltonian

The hamiltonian of $N$ conduction electrons can be expressed, to a good approximation,

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as

$$H(N) = -\frac{\hbar^2}{2m} \sum_i^N \nabla_i^2 + \sum_{i<j} V(r_{ij}) + V'(N), \tag{1}$$

where $m$ is the mass of an electron, $V(r_{ij})$ is the *central force potential* experienced by two electrons and $V'(N)$ stands for the sum of all possible interactions such as electron-phonon, spin-spin, spin-lattice, *etc.* We assume that different components of $V'(N)$ can be treated as perturbation on the states of

$$H_o(N) = H(N) - V'(N). \tag{2}$$

which we identify as a universal component (*i.e.*, independent of the specific nature of a chosen superconductor or a class of superconductors) of $H(N)$. This breakup has an advantage that the impact of *one* (or more than one) component(s) of $V'(N)$ present in a chosen superconductor (or a class of superconductors) can be examined as a perturbation on the states of $H_o(N)$. To find the states of $H_o(N)$, we assume that the electron fluid is a Fermi fluid where $V(r_{ij})$ is the sum of a short range strong repulsion $V^R(r_{ij})$ and an *indirectly induced* weak attraction $V^A(r_{ij})$ of slightly longer range.

To a good approximation, $V^R(r_{ij})$ can be equated to a *hard core* (HC) interaction $V_{HC}(r_{ij})$ defined by $V_{HC}(r_{ij} < \sigma) = \infty$ and $V_{HC}(r_{ij} \geq \sigma) = 0$ where $\sigma$ is the HC diameter of an electron. One finds enough reasons to justify $V^R(r_{ij}) \approx V_{HC}(r_{ij})$, *e.g.*: (i) the screening effect of the lattice greatly reduces the strength and range of inter-electron repulsion, (ii) the conduction electrons flow through certain types of channels *viz.*, a cylindrical tube of diameter $d_c$ in the lattice or a 2-D slot of width $d_c$ between two parallel atomic planes; $d_c$ being a small fraction of lattice constant $a$ speaks about the smallness of their $\sigma$, and (iii) the density of conduction electrons renders inter-electron distance ranging from a vaue $> a/2$ (*cf.* Section 3.4.5) in systems with their number density higher than that of atoms/molecules) to a couple of $a$ in systems (where the said density is lower) and this indicates about the smallness of the range of $V^R(r_{ij})$.

Since no conduction electron comes out of a solid unless a definite amount of energy ($\approx$ work function) is supplied from outside, there are certain factors, such as polarizability of lattice constituents, presence of $+ve$ charges in the background of mobile electrons, *etc.*, which bind each electron with the entire system (the lattice + conduction electrons) indicating the presence of $V^A(r_{ij})$ (*i.e.* the electron-lattice interaction leading to an indirect inter-electron attraction). To a good approximation, $V^A(r_{ij})$ can be replaced by a constant negative potential $-V_o$ whose main role is to keep electrons within the volume of the conductor. This indicates that each conduction electron, to a good approximation, can be identified as a freely moving HC particle on the surface of a constant $-ve$ potential.

## 2.2. Basic unit of the fluid

In what follows from the above discussion, the motion of each conduction electron, to a good approximation, can be expressed by a plane wave

$$u_{\mathbf{p}}(\mathbf{b}) = A \exp(i\mathbf{p}.\mathbf{b}) \tag{3}$$

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where $\mathbf{p}$ and $\mathbf{b}$, respectively, represent the momentum (in wave number) and position vectors of an electron. However, the plane wave motion is modified by its collision with other electron(s) or the lattice structure. A collision could either be a two body collision (electron-electron collision), or a many body collision (*e.g.*, two mutually colliding electrons also collide simultaneously with other electron(s) or lattice structure). In the former case two electrons (say, e1 and e2) simply exchange their momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ or positions $\mathbf{b}_1$ and $\mathbf{b}_2$ without any difference in the sum of their pre- and post-collision energies. However, in the latter case e1 and e2 could be seen to jump from their state of $\mathbf{p}_1$ and $\mathbf{p}_2$ to that of different momenta $\mathbf{p}_1'$ and $\mathbf{p}_2'$ (possibly of different energy) but it is clear that to a good approximation they remain in one of the possible states of two HC particles moving in the absence of other electron(s) and/or lattice. This implies that the complex dynamics of the electron fluid can be described to a good approximation in terms of the simple dynamics of a pair of HC particles (discussed in Section-3 below) as its *basic unit*.

### 3.0 Dynamics of Two HC Particles

### 3.1. Schrödinger equation

The Schrödinger equation of two HC impenetrable particles can be described by

$$\left(-\frac{\hbar^2}{2m}\sum_i^2 \bigtriangledown_i^2 + V_{HC}(r)\right)\psi(1,2) = E(2)\psi(1,2) \tag{4}$$

While the dynamics of two electrons in a many body collision involving lattice structure can be expected to encounter an interaction different from that involved in a two electron or many electron collision but the fact, that the end result of any such collision is to take two electrons from their state of $\mathbf{p}_1$ and $\mathbf{p}_2$ to that of $\mathbf{p}_1'$ and $\mathbf{p}_2'$, indicates that the difference is *unimportant* and we can proceed with our analysis of $\psi(1,2)$ and use its results to find how each electron in $\psi(1,2)$ state assumes a phonon induced inter-electron attraction seemingly essential for the occurrence of superconductivity in widely different superconductors.

The process of solving Eqn. 4 is simplified by using: (i) $V_{HC}(r) \equiv A\delta(r)$ where $A$ representing the strength of Dirac delta function $\delta(r)$ is such that $A \to \infty$ when $r \to 0$ (this type of equivalence has been mathematically demonstrated by Huang [47] and physically argued in Section 3.2, below), and (ii) the *center of mass* (CM) coordinate system defined by,

$$\mathbf{r} = \mathbf{b}_2 - \mathbf{b}_1 \quad \text{and} \quad \mathbf{k} = \mathbf{p}_2 - \mathbf{p}_1 = 2\mathbf{q}, \tag{5}$$

with $\mathbf{r}$ and $\mathbf{k}$, respectively, representing the relative position and relative momentum of two electrons, and

$$\mathbf{R} = (\mathbf{b}_2 + \mathbf{b}_1)/2 \quad \text{and} \quad \mathbf{K} = \mathbf{p}_2 + \mathbf{p}_1, \tag{6}$$

with $\mathbf{R}$ and $\mathbf{K}$, similarly, referring to the position and momentum of their CM. Without loss of generality, Eqns. 5 and 6 also render

$$\mathbf{p}_1 = -\mathbf{q} + \frac{\mathbf{K}}{2} \quad \text{and} \quad \mathbf{p}_2 = \mathbf{q} + \frac{\mathbf{K}}{2}. \tag{7}$$

By using these equations, one may express Eqn. 4 as

$$\left(-\frac{\hbar^2}{4m}\nabla_R^2 - \frac{\hbar^2}{m}\nabla_r^2 + A\delta(r)\right)\Psi(r,R) = E(2)\Psi(r,R) \qquad (8)$$

with

$$\Psi(r,R) = \psi_k(r)\exp(i\mathbf{K}.\mathbf{R}). \qquad (9)$$

We note that the HC interaction affects only the relative motion of two particles $[\psi_k(r)]$ which represents a solution of

$$\left(-\frac{\hbar^2}{m}\nabla_r^2 + A\delta(r)\right)\psi_k(r) = E_k\psi_k(r) \qquad (10)$$

with $E_k = E(2) - \hbar^2 K^2/4m$, while the CM motion $[\exp(i\mathbf{K}.\mathbf{R})]$ remains unaffected.

## 3.2. Basis for $V_{HC}(r) \equiv A\delta(r)$

The physical basis for $V_{HC}(r) \equiv A\delta(r)$ can be understood by examining the possible configuration of two HC particles (say P1 and P2) right at the instant of their collision. When P1 and P2 during a collision have their individual CM located, respectively, at $r_{CM}(1) = \sigma/2$ and $r_{CM}(2) = -\sigma/2$ (with $r_{CM}$ being the distance of the CM of a particle from the CM of P1 and P2), they register their physical touch at $r = 0$ and their encounter with $V_{HC}(r)$ is a result of this touch beyond which two HC particles can not be pushed in. The process of collision only identifies this touch; it does not register how far are the CM points of individual particles at this instant. In other words the rise and fall of the potential energy of P1 and P2 during their collision at $r = 0$ is independent of their $\sigma$ and this justifies $V_{HC}(r) \equiv A\delta(r)$. It may, however, be mentioned that this equivalence will not be valid in accounting for certain physical aspects of the system (*e.g.*, the volume occupied by a given number of particles) where the real size of the particle assumes importance.

## 3.3. Statefunctions

In order to find the statefunction $\Psi(r,R)$, -a solution of Eqn. 8, we treat $A\delta(r)$ as a step potential. Since P1 and P2 experience zero interaction in the region $r \neq 0$, they can be represented, independently, by plane waves except around $r = 0$ where $A\delta(r) = \infty$. However, in view of the possible superposition of two waves, the state of P1 and P2 can, in principle, be described by

$$\Psi(1,2)^\pm = \frac{1}{\sqrt{2}}[u_{\mathbf{p}_1}(\mathbf{r}_1)u_{\mathbf{p}_2}(\mathbf{r}_2) \pm u_{\mathbf{p}_2}(\mathbf{r}_1)u_{\mathbf{p}_1}(\mathbf{r}_2)]. \qquad (11)$$

But we find that $\Psi(1,2)^+$ (of +ve symmetry for the exchange of two particles) does not represent the *desired* wave function of two HC particles since, as required, it does not vanish at $\mathbf{r}_1 = \mathbf{r}_2$ where $A\delta(r=0) = \infty$, while the other function $\Psi(1,2)^-$ of $-ve$ symmetry has no such problem. We addressed this problem in our recent analysis of the 1-D analogue of Eqn. 8 in relation to our detailed study of the wave mechanics of two HC impenetrable particles in 1-D box [41]. In what follows from this study [41] one may easily find that the state of two such particles can be expressed by

$$\zeta(r,R)^\pm = \zeta_k(r)^\pm \exp(i\mathbf{K}.\mathbf{R}) \qquad (12)$$

with

$$\zeta_k(r)^- = \sqrt{2}\sin{(\mathbf{k}.\mathbf{r}/2)} \tag{13}$$

of $-ve$ symmetry, and

$$\zeta_k(r)^+ = \sqrt{2}\sin{(|\mathbf{k}.\mathbf{r}|/2)} \tag{14}$$

of $+ve$ symmetry for the exchange of their $\mathbf{r}_1$ and $\mathbf{r}_2$ (or $\mathbf{k}_1$ and $\mathbf{k}_2$). It is obvious that in a given state of the pair only $\mathbf{r}$ is variable; any change in $\mathbf{k}_1$ and $\mathbf{k}_2$ and/or the angle between $\mathbf{k}$ and $\mathbf{r}$ implies a change in the state. We note that the second derivative of $\zeta_k(r)^+$ with respect to $r$ has $\delta-$like singularity at $r = 0$ which, however, can be reconciled for the presence of infinitely strong repulsive potential at $r = 0$.

### 3.4 Characteristic Aspects

3.4.1. *Nature of relative motion :* We note that $\zeta_k(r)^\pm$, describing the relative motion of two HC particles, is a kind of *stationary matter wave* (SMW) which modulates the probability $|\zeta_k(r)^\pm|^2$ of finding two particles at their relative phase position $\phi = \mathbf{k}.\mathbf{r}$ in the $\phi-$space. Interestingly, the equality $|\zeta_k(r)^-|^2 = |\zeta_k(r)^+|^2$ concludes an *important fact* that the relative configuration and relative dynamics of two HC particles is independent of their fermionic or bosonic nature. This implies that the requirement of fermionic symmetry of electrons should be enforced on the wave functions of their $\mathbf{K}-$motions or spin motions and we use this inference in constructing $N-$electron wave function in Section-4. In agreement with Eqn. 7, the SMW character of $\zeta_k(r)^\pm$ also reveals that two HC particles in $\zeta(r,R)^\pm$ state have equal and opposite momenta $(\mathbf{q},$-$\mathbf{q})$ in the frame attached to their CM which moves with momentum $\mathbf{K}$ in the laboratory frame. It is also evident that the two particles in their relative motion maintains a center of symmetry at their CM (the point of their collision) which implies

$$\mathbf{r}_{CM}(1) = -\mathbf{r}_{CM}(2) = \frac{\mathbf{r}}{2} \quad \text{and} \quad \mathbf{k}_{CM}(1) = -\mathbf{k}_{CM}(2) = \mathbf{q} \tag{15}$$

where $\mathbf{r}_{CM}(i)$ and $\mathbf{k}_{CM}(i)$, respectively, refer to the position and momentum of $i-$th particle with respect to the CM of two particles.

3.4.2. *MS and SS states :* Since $\zeta(r,R)^\pm$ is a result of the superposition of two plane waves of momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ and the state is basically an eigenstate of the momentum/energy operators of the relative and CM motions of two particles in superposition (*not of individual particles*), it could be rightly identified as a state of *mutual superposition* (MS) of two particles. However, one may have an alternative picture by presuming that each of the two particles after their collision falls back on the pre-collision side of $r = 0$ (the point of collision) and assumes a kind of *self superposition* (SS) (*i.e.*, the superposition of pre- and post-collision states of one and the same particle). Interestingly, this is also described by $\zeta(r,R)^\pm$ because it also represents a superposition of the plane wave of momentum $\mathbf{p}_1$ (the pre-collision state of P1) and a similar wave of momentum $\mathbf{p}_1' = \mathbf{p}_2$ representing post-collision state of P1 because two particles exchange their momenta on their collision; the same effect can be seen with P2. However, since P1 and P2 are identical particles and we have no means to ascertain whether they exchange their positions or bounce back after exchanging their momenta, we can use $\zeta(r,R)^\pm$ to identically describe the MS state

of two particles or the SS states of individual particle and this helps us in developing the *macro-orbital representation* of each electron in the fluid (*cf.*, point 3.4.7).

3.4.3. *Values of $< r >$, $< \phi >$ and $< H(2) >$ :* The SMW waveform, $\zeta_k(r)^\pm$, has series of antinodal regions between different nodal points at $r = \pm n\lambda/(2\cos\theta)$ (with $n$= 0,1,2,3, ... and $\theta$ being the angle between $\mathbf{q}$ and $\mathbf{r}$). This implies that two particles can be trapped on the $r$ line *without disturbing their energy or momenta* by suitably designed cavity of impenetrable infinite potential walls. For example, one may possibly use two pairs of such walls and place them at suitable points perpendicular to $\mathbf{k}_1$ and $\mathbf{k}_2$ or to the corresponding $\mathbf{k}$ and $\mathbf{K}$). In case of $\mathbf{k}||\mathbf{r}$ (representing a $s-$wave state) one can use a cavity of only two such walls placed at the two nodal points located at equal distance on the opposite sides of the point ($r = 0$) of their collision. Using the fact that the shortest size of this cavity can be only $\lambda$, we easily find

$$< r >_o \quad = \frac{< \zeta_k(r)^\pm |r| \zeta_k(r)^\pm >}{< \zeta_k(r)^\pm || \zeta_k(r)^\pm >} = \frac{\lambda}{2} \tag{16}$$

as the shortest possible $< r >$. To this effect, integrals are performed between $r = 0$ (when the two particles are at the center of cavity) to $r = \lambda$ (when one particle reaches at $r = \lambda/2$ and the other at $r = -\lambda/2$ representing the locations of the two walls which reflect the particles back inside the cavity). Following a similar analysis for the general case we identically find $< r > = \lambda/(2\cos\theta)$ which not only agrees with Eqn. 16 but also reveals that the two particles assume $< r >=< r >_o$ only when they have head-on collision. Evidently, from an experimental view point, two HC particles never reach closer than $\lambda/2 = \pi/q$ and in this situation their individual locations (*cf.* Eqn. 15) are given by $< \mathbf{r}_{CM}(1) >_o= - < \mathbf{r}_{CM}(2) >_o= \lambda/4$. Finding similar result for their shortest possible distance in $\phi-$space and $< V_{HC}(r) >$, *etc.* we note that $\zeta_k(r)^\pm$ state is characterized by

$$< \zeta_k(r)^\pm |r| \zeta_k(r)^\pm > \quad \geq \quad \lambda/2 \quad \text{and} \quad < \psi_k(r)^\pm |\phi| \psi_k(r)^\pm > \quad \geq 2\pi, \tag{17}$$

$$< \zeta_k(r)^\pm |V_{HC}(r)| \zeta_k(r)^\pm > \quad = \quad < \zeta_k(r)^\pm |A\delta(r)| \zeta_k(r)^\pm > \quad = 0, \tag{18}$$

$$E(2) = \quad < \zeta(r, R)^\pm |H(2)| \zeta(r, R)^\pm > \quad = \left[ \frac{\hbar^2 k^2}{4m} + \frac{\hbar^2 K^2}{4m} \right]. \tag{19}$$

It is evident from these equations that two particles in $\zeta(r, R)^\pm$, basically, have kinetic energy. We note that the fact, that SMW state ((*cf.* Section 3.4.1) does not allow two HC particles to occupy a common point in real space (two particles always stay on the opposite sides of their CM), provides a strong basis to justify Eqn. 18. We analyze Eqn. 18 for its general validity in Appendix-A and we find that it is valid for all physically relevant situations of two particles. We also note that $\zeta(r, R)^\pm$ is not an eigenstate of the momentum/energy operators of individual particle. In stead, it is the eigenstate of the energy operator of the pair of particles which, naturally, share $E(2)$ equally.

3.4.4. *quantum size :* In what follows from Eqns. 17 and 18, a HC particle of momentum $q$ exclusively occupies $\lambda/2$ space if $\lambda/2 > \sigma$ because only then the two particles maintain $< r >\geq \lambda/2$. We call $\lambda/2$ as quantum size of the particle. One may also identify quantum size as the size of a particle (say P1) as seen by the other particle (say P2) or *vice versa* in $\zeta_k(r)^\pm$ state of their wave superposition. To this effect we may consider P1 as an object to

be probed and P2 as a probe (or *vice versa*) and apply the well known principle of image resolution. We find that P2 can not resolve the $\sigma$ size of P1 if $\lambda/2 > \sigma$ and the effective size of P1 as seen by P2 (or *vice versa*) would be limited to $\lambda/2$. But the situation is different for the particles of $\lambda/2 \leq \sigma$ because here they can resolve the $\sigma$ size of each other and P1 and P2 would see each other as particles of size $\sigma$ in all states of $q \geq h/2\sigma$. This concludes that the effective size of low momentum particles ($q < h/2\sigma$) is $q$−dependent, while that of high momentum particles ($q \geq h/2\sigma$) is $q$−independent; this renders an important aspect which can explain why many body systems exhibit the impact of wave nature only at low temperatures.

On the qualitative scale our meaning of "quantum size" seems to be closer to what Huang [47] refers as "quantum spread" but on the quantitative scale, while we relate "quantum size" of a particle with its momentum by a definite relation $\lambda/2 = \pi/q$, "quantum spread" has not been so related. The fact, that no particle can be accommodated in a space shorter than $\lambda/2$, implies that "quantum size" could be identified as the minimum of quantum spread of a particle or as the minimum possible size of space exclusively occupied by the particle. It may also be mentioned that our meaning of "quantum size" differs from that of *quantum size* word in "quantum size effects" on the properties of thin films, and small clusters of atoms [48], *etc.*

3.4.5. *zero-point force :* In what follows from the above discussion, each HC particle in a fluid exclusively occupies a minimum space of size $\lambda/2$ which, obviously, increases with fall in $T$. Evidently, at certain $T = T_o$, at which the average $\lambda/2$ equals $d$ (*the average nearest neighbor distance*), almost all particles have their minimum possible spread and they find themselve in the ground state of a box of size $d$ (a cavity formed by neighboring atoms) with their momenta frozen at $q = q_o = \pi/d$. Using the thermal de Broglie wavelength $\lambda_T = h/\sqrt{2\pi m k_B T}$ as average $\lambda$, we have

$$T_o = \frac{h^2}{8\pi m k_B d^2}. \tag{20}$$

Evidently, each particle at $T \leq T_o$ tends to have $\lambda/2 > d$ by expanding the cavity size; for this purpose it exerts its zero-point force $f_o = h^2/4md^3$ against inter-particle force ($f_a$) that decides the size and structure of the cavity. This happens also to conduction electrons constrained to move through narrow channels (*e.g.*, a cylindrical tube or 2-D slot between two parallel lattice plains in the systems like HTC superconductors) of diameter or width $d_c$ and they all exert $f_o$ on the lattice against its $f_a$ deciding $d_c$. Consequently, lattice has non-zero mechanical strain which plays a crucial role for superconductivity (*cf.* Section 6.0). In this cotext an estimate of shortest $d = (v/n)^{1/3}$ (with $v$ = unitcell volume per atom and $n$ = number of conduction electrons cotributed by the atom) renders $d = a/2$ by using $n = 8$ (the maximum possible $n$). Since even this $d$ is larger than an expected $d_c$, the lowest possible $q = q_o$ for an electron should be decided by $d_c$ which means that $d_c$ is more relevant than $d$ to determine the ground state properties of electron fluid.

3.4.6. *Phase correlation :* Applying the standard procedure [49, 50] for determining the quantum correlation potential $U(\phi)$ between two particles of a many body system, we easily find that two particles have a $\phi$−correlation defined by

$$U(\phi) = -k_B T \ln |\zeta_k(r)^{\pm}|^2 = -k_B T \ln [2 \sin^2(\phi/2)] \tag{21}$$

which represents a kind of binding between them in the $\phi-$space. We note that $U(\phi)$ has a series of $\phi-$points at which it has minimum value ($= -k_B T_o \ln 2$ at $\phi = (2n + 1)\pi$ with $n$ being an integer) and maximum value ($= \infty$ at $\phi = 2n\pi$) which implies that wavemechanical superposition of two particles tries to arrange them in $\phi-$space at the points of minimum $U(\phi)$ which are separated by $\Delta\phi = 2n\pi$. The experimentally observed coherence in the motion of electrons particularly in their superconducting state is a consequence of this fact. It may be mentioned that $T$ in Eqn. 21 should be replaced by $T_o$ (Eqn 20 with $d = d_c$) representing $T$ equivalent of $\varepsilon_o = h^2/8md_c^2$ because $q-$motion energy of each electron at $T \leq T_o$ gets frozen at $\varepsilon_o$ (cf. Section 4.2).

3.4.7. *Macro-orbital representation :* We note that in spite of their binding in the $\phi-$space as concluded above, two HC particles in the real space experience a kind of mutual repulsion, if they have $< r >< \lambda/2$, or no force, if $< r >\geq \lambda/2$. This implies that each particle in $\zeta(r, R)^\pm$ pair state can be identified as independent particle in its self superposition (cf. point 3.4.2) represented by a kind of pair waveform $\xi \equiv \zeta^\pm(r, R)$ proposed to be known as macro-orbital and expressed as,

$$\xi_i = \sqrt{2}\sin[(\mathbf{q}_i.\mathbf{r}_i)]\exp(\mathbf{K}_i.\mathbf{R}_i), \tag{22}$$

where $i$ ($i = 1$ or $2$) refers to one of the two particles; here $r_i$ could be identified with $r_{CM}(i)$ (cf. Eqn. 15) which varies from $r_i = 0$ to $r_i = \lambda/2$, while $R_i$ refers to the CM point of $i-$th particle. Although, two particles in $\zeta(r, R)^\pm$ state are independent but it is clear that each of them represents a ($\mathbf{q}$, -$\mathbf{q}$) pair whose CM moves with momentum $\mathbf{K}$ in the lab frame. This implies that each particle in its macro-orbital representation has two motions: (i) the plane wave $K-$motion which remains unaffected by inter-particle interactions, and (ii) the $q-$motion, affected by the inter-particle interaction. In other words a macro-orbital identifies each electron as a particle of effective size $\lambda/2$ moving with momentum $K$ and this gives due importance to the quantum size of a quantum particle or equivalently to the wave packet (again of size $\lambda/2$) manifestation of a quantum particle as invoked by wave mechanics. We find that this picture is consistent with two fluid phenomenology of superconductivity (cf. Section 7.2). Since $\zeta(r, R)^\pm$ is neither an eigenfunction of the energy operator nor of the momentum operator of a *single particle*, each particle shares the pair energy $E(2)$ equally. We have

$$E_1 = E_2 = \frac{E(2)}{2} = \frac{\hbar^2 q^2}{2m} + \frac{\hbar^2 K^2}{8m} \tag{23}$$

It is interesting to note that two particles, having different momenta ($\mathbf{p}_1$ and $\mathbf{p}_2$) and different energy before their superposition, have equal momentum/energy in $\zeta(r, R)^\pm$ state which indicates that their wave superposition take them into a kind of degenerate state and this tends to happen with all electrons when the system is cooled through certain $T = T_a$ lower than $T_o$ (cf. Section 6.1). In order to show that $\xi_i$ fits as a solution of Eqn. 4 [with $V_{HC}(r) \equiv A\delta(r)$], we recast the two particle hamiltonian $H_o(2) = -\sum_i^2(\hbar^2/2m)\nabla_i^2 + A\delta(r)$ as $H_o'(2) = \sum_i^2 h(i) + A\delta(r)$ by defining

$$h_i = -\frac{\hbar^2}{2m}\nabla_i^2 \quad \text{and} \quad h(i) = \frac{h_i + h_{i+1}}{2} = -\frac{\hbar^2}{8m}\nabla_{R_i}^2 - \frac{\hbar^2}{2m}\nabla_{r_i}^2 \tag{24}$$

with $h_{N+1} = h_1$ for a system of $N$ particles. It is evident that $\xi_i$ is an eigenfunction of $h(i)$ with $< h(i) >= (\hbar^2 q_i^2/2m + \hbar^2 K_i^2/8m)$ and the two particle wave function, $\Phi(2) = \xi_1\xi_2$

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(or with added permuted terms), is an eigenfunction of $H'_o(2)$ with $< H'_o(2) >= E(2)$ (*cf.* Eqn. 19) because $< A\delta(r) >= A|\xi_1|^2_{r_1=0}|\xi_2|^2_{r_2=0} = A\sin^2 q_1 r_1|_{r_1=0} \sin^2 q_2 r_2|_{r_2=0} = 0$ since $r = 0$ implies $r_1 = r_2 = 0$ (*cf.* Eqn. 15).

3.4.8. *Accuracy and relevance of macro-orbitals* : While the fact, that the fall of an electron into its SS state (*cf.*, Section 3.4.2) is independent of the details of the collision (*i.e.*, two body collision, many body collision or the collision with the lattice structure), justifies its representation by $\xi_i$ in general, we also find that the functional nature of $\xi_i$ matches almost exactly with

$$\eta_{q,K}(s, Z) = A\sin[(\mathbf{q.s})]\exp(\mathbf{K.Z}) \tag{25}$$

representing a state of a particle in a cylindrical channel with $\mathbf{s}$ being the 2-D space vector perpendicular to $z-$axis (the axis of the channel) and,

$$\eta_{q,K}(z, S) = B\sin[(\mathbf{q.z})]\exp(\mathbf{K.S}) \tag{26}$$

which represents a similar state of a particle trapped between two parallel impenetrable potential sheets. Interestingly, since superconductivity is a behavior of low energy electrons and a conduction electron in a solid can be visualized, to a good approximation, as a particle moving along the axis of cylindrical channel (*e.g.* in a conventional superconductor) or that moving between two parallel atomic sheets (*e.g.* in HTC systems), the accuracy and relevance of macro-orbitals in representing the conduction electrons in their low energy states is well evident.

## 4.0 States of $N-$Electron Fluid

### 4.1 General state

Using $N$ macro-orbitals for $N$ electrons and following standard method, we have

$$\Psi^j_n(N) = \Pi^N_i \zeta_{q_i}(r_i) \sum_P^{N!} (\pm 1)^P \Pi^N_i \exp\left[i(P\mathbf{K}_i\mathbf{R}_i)\right] \tag{27}$$

for one of the $N!$ microstates of the system of energy $E_n$ (*cf.* Eqn. 29, below). Here $\sum_P^{N!}$ represents the sum of $N!$ product terms obtainable by permuting $N$ particles on different $\mathbf{K}_i$ states with $(+1)^P$ and $(-1)^P$, respectively, used for selecting a symmetric and anti-symmetric wave function for an exchange of two particles. In principle, the permutation of $N$ particles on different $\mathbf{q}_i$ states renders $N!$ different $\Psi^j_n(N)$ and we have

$$\Phi_n(N) = \frac{1}{\sqrt{N!}} \sum_j^{N!} \Psi^j_n(N) \tag{28}$$

as the complete wave function of a possible quantum state of energy $E_n$ given by

$$E_n = \sum_i^N \left[\frac{\hbar^2 q_i^2}{2m} + \frac{\hbar^2 K_i^2}{8m}\right] \tag{29}$$

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where $q_i$ and $K_i$ can be an integer multiple of $\pi/d_c$ and $\pi/L$, respectively. To follow Eqn. 29, one may use Eqn. 24 to recast $H_o(N) \approx \sum_i^N h_i + \sum_{i>j}^N A\delta(r_{ij})$ as

$$H_o(N) = \sum_i^N h(i) + \sum_{i>j}^N A\delta(r_{ij}) \tag{30}$$

In what Follows from Eqn. 29, the energy of conduction electrons is basically kinetic which seems to imply that these electrons constitute a system of some kind of non-interacting fermions, while this is not true. This *apparent result* is obtained because $< V_{HC}(r_{ij}) >= 0$ (*cf.* Eqn. 18) which agrees with the fact that two electrons do not occupy common point in real space. In addition one may find that the presence of $V_{HC}(r_{ij})$ restricts $< r >$ through $< r >\geq \lambda/2$ (Eqn. 17) (or $q$ through $q \geq q_o$) which clearly indicates that $V_{HC}(r_{ij})$ plays an important role in deciding the relative configuration (*i.e.* the allowed $< r >$, $< \phi >$ and $q$) of conduction electrons, particularly, when the system assumes the ground state of their $q-$motions with all electrons having $q = q_o$.

## 4.2 Ground state

We note that each conduction electron has two motions $q$ and $K$. While the $q-$motions are constrained to have $q \geq q_o(= \pi/d_c)$, -the lowest possible $q$ of a particle restricted to move through channels of size $d_c$, the $K-$motions are guided by the Pauli exclusion principle. Consequently, the ground state of the fluid is defined by all $q_i = q_o$ and different $K_i$ ranging between $K = 0$ to $K = K_F$ (the Fermi wave vector) which render

$$E_{GSE} = N\varepsilon_o + \bar{E}_K = N\frac{h^2}{8md_c^2} + \frac{1}{4}\cdot\frac{3}{5}NE_F \tag{31}$$

as the *ground state energy* of the fluid. Here $\varepsilon_o = h^2/8md_c^2$ represents lowest possible energy of the $q-$motion of an electron and $\bar{E}_K$ being the net $K-$motion energy of $N$ electrons with $E_F$ being the Fermi energy; the factor $1/4$ in the last term represents the fact that each electron in its macro-orbital representation behaves like a particle of mass $4m$ for its $K-$motions. In order to understand how different inter-particle interactions enter in our formulation to control the ground state energy of electron fluid, it is important to note that $d_c$ in a given system is decided by all such interactions. Naturally, all these interactions indirectly control the ground state momentum through $q_o = \pi/d_c$ and hence the ground state energy $\varepsilon_o$. Expressing $E_{GSE}$ (Eqn. 31) in terms of its temperature equivalent, we have

$$T_{GSE} = T_o + T(\bar{E}_K) \approx T_o + 0.15T_o \tag{32}$$

where we use $T_o \equiv \varepsilon_o$ and $T(\bar{E}_K) \equiv 3E_F/20)$. In writing $T(\bar{E}_K) = 0.15T_o$ we approximated $E_F(\approx h^2/8md^2)$ to $\varepsilon_o(= h^2/8md_c^2)$ by using $d_c$ for $d = (V/N)^{1/3}$ where V is the net volume of the solid containing $N$ electrons. Since $d$ is always expected to be larger than $d_c$, $T = 1.15T_o$ (Eqn. 32) can be identified as the upper bound of $T_{GSE}$, while $T_o$ being the lower bound.

We note that: (i) each conduction electron in the ground state is expressed by a stabilized macro-orbital, $\sin(q_o r) \exp i(\mathbf{K.R})$, for which we easily have $< k >=< -i\hbar\nabla_r >= 0$ by using the fact that $r$ varies between $r = 0$ to $r = d_c$ and (ii) $< r >$ of each conduction

electron lies on the axis of the cylindrical tube (the channel through which they move). While inference-(i) concludes that for all practical purposes two conduction electrons cease to have relative momentum indicating loss of collisional motion or scattering with other electrons or lattice (with $q = q_o$ simply representing the momentum of their localization), inferences-(i and ii) reveal that conduction electrons can move (*if they are set to move*) only in the order of their locations in the channel(s), obviously, with identically equal momentum, -a characteristic of coherent motion. Since the conduction electrons are the constituents of the solid, their real space positions (at least in the ground state) have to be compatible with the crystal structure.

## 5.0 Equation of State

What follows from Eqn. 29, the energy of a particle in our system can be express as

$$\epsilon = \varepsilon(K) + \varepsilon(k) = \frac{\hbar^2 K^2}{8m} + \frac{\hbar^2 k^2}{8m}. \tag{33}$$

However, since the lowest $k = 2q$ is restricted to $2q_o$ for the condition, $q \geq q_o$, $\epsilon$ can have any value between $\varepsilon_o = \hbar^2 q_o^2/8m$ and $\infty$. Interstingly, this possibility exists even if $\hbar^2 k^2/8m$ in Eqn. 33 is replaced by the lowest energy $\varepsilon_o$ since $K$ can have any value between 0 and $\infty$. In other words, we can use

$$\epsilon = \frac{\hbar^2 K^2}{8m} + \varepsilon_o \tag{34}$$

which is valid, to a very good approximation, at low temperatures where we intend to study the system. Using Eqn. 34 in the starting expressions of the standard theory of a system of fermions [51, Ch.8] we obtain

$$\frac{PV}{k_B T} = -\Sigma_{\varepsilon(K)} \ln \left[1 + z \exp\left(-\beta[\varepsilon(K) + \varepsilon_o]\right)\right] \tag{35}$$

and

$$N = \Sigma_{\varepsilon(K)} \frac{1}{z^{-1} \exp\left(\beta[\varepsilon(K) + \varepsilon_o]\right) + 1} \tag{36}$$

with $\beta = \frac{1}{k_B T}$ and fugacity

$$z = \exp\left(\beta\mu\right) \qquad (\mu = \text{chemical} \quad \text{potential}). \tag{37}$$

Once again, by following the steps of the standard theory [51] and redefining the fugacity by

$$z' = z \exp\left(-\beta\varepsilon_o\right) = \exp\left[\beta(\mu - \varepsilon_o)\right] = \exp\left[\beta\mu'\right] \qquad \text{with} \quad \mu' = \mu - \varepsilon_o \tag{38}$$

we easily have

$$\frac{P}{k_B T} = -\frac{2\pi(8mk_B T)^{3/2}}{h^3} \int_0^\infty x^{1/2} \ln\left(1 - z'e^{-x}\right)dx = \frac{g}{\lambda^3} f_{5/2}(z') \tag{39}$$

and

$$\frac{N}{V} = \frac{2\pi(8mk_B T)^{3/2}}{h^3} \int_0^\infty \frac{x^{1/2}dx}{z'^{-1}e^x - 1} = \frac{g}{\lambda^3} f_{3/2}(z') \tag{40}$$

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where $g$ is the weight factor that arises from inherent character such as spin of particles, $x = \beta\varepsilon(K)$, $\lambda_T = h/(2\pi(4m)k_BT)^{1/2}$ and $f_n(z')$ has its usual expression. This reduces our problem of HC particles to that of non-interacting fermions but with a difference. We have $m$ replaced by $4m$ and $z$ by $z'$ (or $\mu$ by $\mu' = \mu - \varepsilon_o$). The range of $z$ and $z'$ remain unchanged. In other words if $\mu$ and $z$ are, respectively, replaced by $\mu'$ and $z'$, system of HC fermions can be treated as a system of non-interacting fermions. As such we can use Eqns. 35 and 36 and Eqns. 39 and 40 to evaluate different thermodynamic properties of our system. For example, the internal energy $U = -\frac{\partial}{\partial\beta}(\frac{PV}{k_BT})|_{z,V}$ of our system can be expressed as,

$$U = \frac{3}{2}k_BT\frac{V}{\lambda^3}f_{5/2}(z') + N\varepsilon_o = U' + N\varepsilon_o \tag{41}$$

with $U' = -\frac{\partial}{\partial\beta}(\frac{PV}{k_BT})|_{z',V}$ being the internal energy contribution of non-interacting quasi-particle fermions representing $K-$motions and $N\varepsilon_o$ being the added contribution from $k-$motions. Similarly, we have Helmholtz free energy

$$A = N\mu - PV = N\varepsilon_o + (N\mu' - PV) = N\varepsilon_o + A' \tag{42}$$

with $A'$ being the Helmholtz free energy of non-interacting fermions. Following standard methodology, we now analyze the free energy $A$ for the physical conditions for which it becomes critical and leads to superconductivity.

## 6.0 Important Aspects of Superconductivity

### 6.1 Free energy and its criticality

The free energy of the electron fluid (Eqn. 42) is a sum of two terms: (i) $A'$ representing the contribution of plane wave $K-$motions which define a system of non-interacting quantum quasi-particles of fermionic symmetry and mass $4m$ and (ii) $N\varepsilon_o$ representing the zero-point energy of $q-$motions. We note that a term like $A'$ alone can represent our system at higher temperatures at which $\lambda$ of electrons at large satisfies $\lambda/2 << d_c$ for which they do not have an effective wave superposition envisaged for their macro-orbital representation and they can be represented, to a good approximation, by plane waves. This observation agrees with the experimental fact that the behavior of electron fluid in solids at high temperatures fits very well with the theory of Fermi gas (non-interacting fermions) available in every text on Fermi statistics, $e.g.$, [47, 51]. Since $A'$ does not become critical at any $T$, the criticality of the electron fluid leading to its superconductivity must arise only with $N\varepsilon_o = Nh^2/8md^2$. However, since $N\varepsilon_o$ has no explicit dependence on the parameters such as temperature $T$, pressure $P$, $etc.$, it provides no explicit mathematical solution for $T_c$, $P_c$, $etc.$ at which $N\varepsilon_o$ becomes critical. It is for this reason that *we examine the system for its criticality by analyzing its evolution on cooling* and using the condition $q \geq q_o$ which makes the system critical when $q$ tend to fall below $q_o$ at certain $T = T_a$.

Since $\lambda_T$ increases on cooling, almost all electrons can have $\lambda/2 = d_c$ (or $q = q_o$) at $T_a$ close to $T_o$ (Eqn. 20 with $d = d_c$). However, Pauli exclusion can be expected to push it down because $q = q_o$ state for all electrons represents a kind of degenerate state which can be disturbed effectively when electrons have enough energy in their $K-$ motions (*cf.*

Section 6.4). Evidently, to have a good estimate of $T_a$ we find the number of conduction electrons $N_{2q_o}(T)$ in $q = 2q_o$ state (the first excited state of $q-$motion) at $T = T_a$ by using $N_{2q_o}(T) = N \exp\left[-(4\varepsilon_o - \varepsilon_o)/k_B T\right]$ as a good approximation. We find $N_{2q_o}(T_o) \approx 10^{-1}N$ which means that only about 90% particles occupy $q = q_o$ state at $T = T_o$ indicating that $T_a < T_o$. However, a similar estimate of $N_{2q_o}(T)$, for $T = 0.15T_o$ which represents the $T$ equivalent of the least amount of $K-$motion energy retained by electrons at $T = 0$, reveals $N_{2q_o}(0.015T_o) \approx 10^{-7}N$ which implies that the percentage of electrons in $q = q_o$ state is 99.99999%. Evidently, the occupancy of $q = q_o$ state by macroscopically large number of electrons seems to reach its completion *for all practical purposes* at a $T > 0.15T_o$. Naturally, electrons can have $q < q_o$ by using their zero-point force to expand the channel size at $T = T_a$ which is expected to fall between $T_o$ and $0.15T_o$. We analyze this process of straining the lattice in the follwing section.

## 6.2 Onset of lattice strain

In what follows from the above analysis, electrons tend to have $q < q_o$ (*i.e.* $\lambda/2 > d_c$) when they are cooled through $T_a$. They occupy more space in the lattice structure by using their zero point force $f_o = h^2/4md_c^3$ against the interatomic forces (say $f_a$) which restore the lattice structure. The equilibrium between $f_o$ and $f_a$, obviously, renders a non-zero strain $\Delta d = d_c' - d_c$ (with new $q = q_o' = \pi/d_c'$) in the lattice and this happens for almost all conduction electrons leading to an onset of the process of lattice straining around $T_a$. The experimental fact, that liquids $^4He$ and $^3He$ on their cooling, respectively, through 2.17K and 0.6K (matching closely with $T_a$ [$0.15T_o < T_a < T_o$ for $T_o \approx 1.4$K) exhibit volume expansion (*i.e.* -ve volume expansion coefficient) [52], proves that $f_o$ (expected to operate around $T_a \leq T_o$) undoubtedly produces strain (expansion) in $He - He$ bonds and there is no reason for which similar effect is not expected from the $f_o$ of conduction electrons in superconductors. In fact the recent experimental studies [44, 45] have confirmed the presence of mechanical strain in HTC systems.

## 6.3 Energy gap and (q, -q) bound pairs

With the onset of lattice strain $\Delta d$ (*cf.* Section 6.2), the $q-$motion energy of a conduction electron falls below $\varepsilon_o$ by

$$\Delta\epsilon = \varepsilon_o - \varepsilon_o' = \frac{h^2}{8md_c^2} - \frac{h^2}{8m(d_c + \Delta d)^2} = \frac{h^2}{4md_c^3}(\Delta d).$$ (43)

In view of the nature of $q-$ and $K-$motions [*cf.* Eqns. 8-10], it is evident that the process of straining the lattice only affects the $q-$motions and $\Delta\epsilon$ represents a decrease in the zero-point energy of this motion only. A simple analysis of the equilibrium between $f_a$ (*cf.* Section 6.2) and $f_o$, concludes [53] that, to a good approximation, half of $\Delta\epsilon$ is *stored with the lattice as its strain energy* ($\epsilon_s$) leaving the rest half

$$\epsilon_g = \frac{h^2}{8md_c^3}(\Delta d).$$ (44)

as the net fall in the energy of an electron in the locally strained lattice. In what follows from a detailed analysis [53] of certain simple representative examples of trapped quantum

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particle(s) interacting with oscillating particle(s), we also find that $q$ of such an electron oscillates with the frequencies of lattice oscillations (*i.e.* phonons). To understand the this inference, without going through the dertails available in [53], it may be noted that zero-point energy ($\varepsilon_o$) and momentum ($q_o$) of a particle constrained to move through some sort of channels in the lattice structure, obviously, depends on the diameter/ width ($d_c$) of the channel. Naturally, when $d_c$ oscillates with a lattice oscillation at a phonon frequency, $\varepsilon_o$ and $q_o$ also oscillate with the same frequency and the electron and lattice can be seen to exchange energy/ momentum from each other [53]. Since an electron remains in this state unless it receives $\epsilon_g$ energy from outside, $\epsilon_g$ can be identified as an *energy gap* between its state with strained lattice and that with zero-strained lattice. Further as each electron in our theoretical framework represents (**q**, -**q**) pair, the existence of this gap means that conduction electrons in the fluid are in a state of (**q**, -**q**) bound pairs and the effective free energy of $q-$motions can be expressed by

$$N\varepsilon'_o = N\varepsilon_o - N\epsilon_g(T) = N\varepsilon_o - E_g(T) \tag{45}$$

where $E_g(T)$ is the net decrease in the free energy of the electron fluid. Since as discussed in Section 2.0, each electron binds with lattice and $N-1$ other conduction electrons, $E_g(T)$ could be identified as the collective binding of all electrons in the solid. However, it is argued (Section 6.4 below), that this gap does not show its effectiveness unless the system is cooled to $T < T_c(\equiv \epsilon_g)$.

## 6.4 Transition temperature

Two electrons (as per Pauli exclusion) can have : (i) different $K$ and equal $q$ or (ii) equal $K$ and different $q$. As a result of the latter possibility many electrons can have $q > q_o$ at the cost of their $K-$motion energy and the state of $q = q_o$ even for $T \leq T_o$ (excluding $T \leq T_c$) does not attain stability. Consequently, inter-electron binding induced by lattice strain does not operate effectively unless the system is cooled to $T \leq T_c(\equiv \epsilon_g)$. This renders

$$T_c = \frac{h^2}{8\pi m k_B d_c^2}\frac{\Delta d}{d_c} = T_o\frac{\Delta d}{d_c} = T_o\frac{\beta l}{d_c} \tag{46}$$

with $T_o = h^2/(8\pi m k_B d_c^2)$, and $l = a$ (representing the *interatomic separation* in conventional superconductors) or $l = c$ (*the lattice parameter perpendicular to the conduction plane of electrons in HTC systems*). In Eqn. 46, we use $\Delta d = \beta l$ in view of the fact that the strain $\Delta d$ should be proportional to $l$ with proportionality constant $\beta$ representing a kind of the elastic property of interatomic bonds (in conventional systems) or the lattice parameter $c$ in HTC systems. It is evident that $T_c$ represents a transition temperature below which the conduction electrons are in a stable state of their $q-$motions. Since these electrons cease to have relative motion (Section 4.2), they move in order of their positions without any collision or scattering which means that they only have correlated motion without disturbing their relative positions in real and phase spaces. The stability of this state is not disturbed by any perturbation of energy $< \epsilon_g$, (*viz.* external magnetic field, electric current *etc.*), which indicates that *the long range electron-electron correlations mediated by phonons* and related properties (superconductivity, coherence, persistence of currents, *etc.*) are also not disturbed unless the energy of these perturbations crosses

its critical magnitude. The observation of critical magnetic field(s), critical currents, *etc.* support this observation.

## 6.5 Nature of transition

The well known fact that $A'$ does not become critical at any $T$ [51] implies that it has no energy change at $T_c$. On the other hand $N\varepsilon_o$ has only marginal change with the process of straining lattice (or the formation of $(\mathbf{q}, -\mathbf{q})$ bound pairs) which starts at $T_a$ and continues till $T = 0$. Evidently, the changes from $N\varepsilon_o(T_a)$ to $N\varepsilon_o(T = 0) = N\varepsilon_o(T_a) - E_g(T = 0)$ lasts over a wide range (from $T_a$ to $T = 0$) of $T$. To this effect it may be noted that electrons assume their bound pair states at $T \approx T_a > T_c$. Naturally, the fall in their energy by $E_g(T = 0)$ has already taken place but because of Pauli exclusion $E_g(T = 0)$ does not assume its effectiveness unless the system is cooled through $T_c$. This clearly means that $N\varepsilon_o$ passes smoothly from $N\varepsilon_o(T_c^+)$ to $N\varepsilon_o(T_c^-)$, *i.e.* without any jump in energy. Evidently, the transformation of the electron fluid into its superconducting state at $T_c$ is a *second order transition*.

When electrons at $T_c$ move from their state of $\lambda/2 < d_c$ (normal fluid state at $T = T_c^+$) to that of $\lambda/2 = d_c$ (superconducting state at $T = T_c^-$), their relative $\phi-$positions ($\phi = kd$) change from $\phi > 2\pi$ to $\phi = 2\pi$. This means that electrons move from their configuration of random locations in $\phi-$space to that of orderly separated $\phi-$positions ($\Delta\phi = 2n\pi$ with $n = 1, 2, 3, ..$). In other words the electrons have a kind of order-disorder transformation in $\phi-$space with their transformation to superconducting state at $T_c$. Evidently, because of this order, electrons in superconducting state maintain a definite phase correlation and exhibit coherence of their motion as well as quantized vortices.

## 6.6 Typical estimates of $T_c$

The universal component of the hamiltonian $H_o(N)$ (Eqn. 2) of electron fluid in a solid does not differ from that of liquid $^3He$ (if spin-spin interaction and spin-orbital interactions are excluded in this case too). Evidently, superfluid $T_c$ for both fluids can be obtained by Eqn. 46. Since the desired experimental data about $d$ and strain $\Delta d$ of reasonably high accuracy are available for liquid $^3He$, it is instructive to determine its $T_c$ from Eqn. 46 and compare the same with experimental value to have an idea about the accuracy of Eqn. 46. Therefore, we use the density data available from [52] to determine (i) $d = 3.935718\mathring{A}$ at $T = 0.6$K at which the volume expansion (or onset of $He - He$ bond strain is observed), (ii) $d = 3.939336\mathring{A}$ at $T = 0.1$K and (iii) $\Delta d = 0.003618\mathring{A}$ to find $T_c = 1.497$mK which agrees very closely with experimental $T_c \approx 1.0$mK [54, 55]. It may be observed that no other theory [56] has predicted a $T_c$ for liquid $^3He$ which falls so close to the experimental value. Evidently, this indicates the accuracy of Eqn. 46.

Although, crystal structural data for widely different superconducting solids are available in the literature, and one can use these data to determine the inter-particle distance but there is no way to find an accurate value of the channel size $d_c$ through which conduction electrons flow and strain $\Delta d_c$ produced by $f_o$ of electrons. Consequently, one can use Eqn. 46 for electron fluid only to estimate the range of typical values of $T_c$ by using typical numbers for $d_c$ and $\Delta d_c$. To this effect we first find that the force constant $C_o = 2.735$ dyne/cm (estimated from $C_o = 3h^2/4md^4$) related to $f_o$ for liquid $^3He$

matches closely with $He-He$ single bond force constant $\approx 2.0$ dyne/cm estimated from zero wave vector phonon velocity 182 m/sec [52]. A similar estimate of $C_o$ for the $f_o$ of electrons can be made by using (i) $d_c = 3.935718\mathring{A}$ (*i.e.* as large as $d_{He-He}$) and (ii) as short as $d_c = 1.0\mathring{A}$ which is expected to represent the typical $d_c$ for superconducting solids. Using standard value of electron mass $m_e = 0.9109 \times 10^{-27}$ gm, we, respectively, found $C_o = 15 \times 10^3$ dyne/cm and $C_o = 36.0 \times 10^5$ dyne/cm which compares well with the typical force constants for a bond between two nearest neighbors in widely different solids. In view of this observation, we assume that the strain factor $\Delta d/d$ in superconducting solids approximately has the same value ($= 9.1897 \times 10^{-4}$) that we observe experimentally for liquid $^3He$ and use Eqn. 46 to find $T_c = 8.23$ K for $d_c = 3.935718\mathring{A}$ and $T_c = 124$ K for $d_c = 1.0\mathring{A}$ which closely fall in the range of experimentally observed $T_c$. Evidently, Eqn. 46 explains the experimentally observed $T_c$ for conventional superconductors as well as HTC systems.

## 6.7 Factors affecting $T_c$

Since conduction electrons in a solid move in an interacting environment, $m$ in Eqn. 46 could be replaced by $m^*$ (the effective mass of the electron). Evidently, $T_c$ depends on channel size $d_c$, strain $\beta l$, and $m^*$ which means that one may, in principle, change $T_c$ at will if there is a method by which these parameters for a given solid can be suitably manipulated. However, any controlled change in these parameters does not seem to be simple. For example we may apply pressure to decrease $d_c$ in order to increase $T_c$ but the compression produced by pressure may increase electron-lattice interactions in such a way that an increase in $m^*$ may overcompensate the expected increase in $T_c$ and one may find that $T_c$ decreases with increase in pressure. Evidently, though $T_c$ is normally expected to increase with pressure, its pressure dependence, for some superconductors, may show opposite trend or complex nature. Similarly, we can take the example of a change in $T_c$ with $\Delta d$ which equals $\beta c$ for a HTC system and $\beta a$ for a conventional superconductor. Since $\beta c$ is much larger than $\beta a$, lattice strain could be one factor which may increase $T_c$ of a HTC system by a factor of $c/a$, if $d_c$, $\beta$, $m^*$, *etc.* for two types of systems do not differ. As analyzed by Leggett [57], $T_c$ increases with the number of conducting planes ($n_{cp}$) per unit cell for certain groups of HTC systems which indicates that $T_c$ realy increases with $c$, since $c$ increases with $n_{cp}$. However, $T_c$ does not increase with $n_{cp}$ always [57] and we find that the dependence of $T_c$ on $d_c$, $\beta$, $m^*$ and $\Delta d$ is not so simple as it appears from Eqn. 46. One may hope that there could be some possible mechanism which may help in raising $T_c$ by manipulating these parameters. In this context, it may be emphasized that Eqn. 46 does not rule out the possibility of achieving *room temperature* (RT) superconductivity since raising $T_c$ from 124K to 300K simply requires a system where $(1/m^* d_c^2)(\Delta d/d_c)$ is increased from 1.0 to 2.5 which can achieved if $m^*$ alone changes from $m$ to $0.4m$ or $\Delta d/d_c$ changes from 0.001 to 0.0025 or $d_c$ is reduced by a factor of 1.6. As a matter of principle any change or perturbation, which adds (*removes*) energy to (*from*) the $q-$motions, will decrease (*increase*) $T_c$.

## 6.8 Strain energy of lattice

We note that the strain in lattice produced by (*say*) $i-$th electron is a local effect and its magnitude depends on the quantum size ($\lambda_i/2$) and hence the momentum ($q_i$) of the

electron which renders $\epsilon_s = \epsilon_s(q_i)$. However, since identical local strains are produced by all conduction electrons distributed uniformly in the solid, a collective long range impact of these strains can be observed due to strong inter-atomic forces, and the net strain energy of the lattice can be expressed as $E_s = E_s(q_1, q_2, q_3, ...)$. Naturally, because of this dependence of $E_s$ on momenta of all conduction electrons, a sustained exchange of energy between electrons and lattice is expected when the channel size oscillating with a lattice oscillation leads the quantum size of different electrons to oscillate with the same frequency.

Considering a simple unit of two electrons (say e1 and e2) separated by a small lattice block between them, it can be easily visualized that two electrons will gain (*lose*) energy from the strained lattice if the said lattice block has a kind of breathing oscillation with an expansion (*contraction*) in its size leading to a decrease (*increase*) in the strain and the strain energy of the said block; this will also render a decrease (*increase*) in the size of two channels occupied by e1 and e2 for which their $\varepsilon_o$ would obviously increase (*decrease*). However, if the position of the said block oscillates around its CM without any change its size, e1 and e2 would exchange energy with each other. If the block moves to wards e1, it decreases $d_c$ for e1 and increases $d_c$ for e2, and in the process $\varepsilon_o(e1)$ increases at the cost of $\varepsilon_o(e2)$ and the necessary energy flows through appropriate mode(s) of phonons from e1 to e2 and vice versa. It is evident that the dynamical motions in a solid are complex and the two motions that we considered in this example could be the simplest possible modes but these examples explain the typical nature of the process of energy exchange between two electrons and lattice through phonons. Since $E_s$ stays with the lattice even at $T = 0$ at which no phonon exists, $E_s$ serves as the source of phonons and supports phonon mediated correlated motion of conduction electrons by sustained energy exchange between electrons and lattice at all $T \leq T_c$ including $T = 0$.

## 6.9 Order parameter(s)

The conduction electrons in their superconducting state are in the ground state of their $q-$motions with free energy $N\varepsilon_o - E_g(T)$. Since $N\varepsilon_o$ is a constant value which depends on the relevant parameters of the solid at $T = T_a$, only $E_g(T)$ is crucial for different aspects of superconducting state. The lattice strain, on which $E_g(T)$ depends, can be identified as the basic order parameter of the transition. However, since the conduction electrons below $T_c$ assume a configuration where they have : (i) some sort of localization in their positions in the real space unless they are set to move, (ii) an ordered structure in $\phi-$space defined by $\Delta\phi = 2n\pi$ with $n = 1, 2, 3, ...$, (iii) definite momentum $q = q'_o$, (iv) definite orientation of their spins *if different interactions involving spins so prefer* (*cf.* Section 6.11), (v) definite amount of superfluid density $\rho_s$ (*cf.* Sections 7.2 and 7.3), etc. Naturally, electrons at $T \approx T_c$ must have large amplitude position fluctuation (leading to charge density fluctuation), $\phi-$fluctuation, momentum fluctuation, spin fluctuation, $\rho_s-$fluctuation, *etc.* which should naturally couple with the lattice strain identified as the basic order parameter of the superconducting transition. However, the nature of coupling may differ from system to system.

## 6.10 Comparison with normal state

Although, each conduction electron in its macro-orbital picture represents a pair of electrons moving with $\mathbf{q}$ and -$\mathbf{q}$ momenta in their CM frame which moves with momentum $\mathbf{K}$ in the laboratory frame, however, as discussed in Section 6.4, Pauli exclusion does not allow this pair to assume stability at $T \geq T_c$. Consequently, like the particles of any normal liquid, electrons in their normal fluid state have random motions and inter-electron and electron-lattice collisions. However, with the onset of mechanical strain in the lattice structure at $T_c$, electrons assume the configuration of ($\mathbf{q}$, -$\mathbf{q}$) bound pairs with well defined binding energy ($\epsilon_g$) and $q = q'_o$ and as discussed in Section 6.3 they have sustained energy exchange with lattice. Similarly they also have sustained $\phi-$correlation (*cf.* Section 6.5) with well defined positions ($\phi = kr = 2qr = 2n\pi$) in $\phi-$space which means that they have a kind of orderly arrangement in real space since they all have $q = q'_o$. In other words the conduction electrons in their superconducting state have sustained $q-$, $r-$ and $\phi-$correlations which are not seen in their normal state. Further as the conduction electrons in their normal state exchange energy with the lattice by way of their collisions with the lattice constituents but such collisions do not exist in the ground state (*cf.* Section 4.2) that exhibits superconductivity.

In view of our inferences made in Section 4.2, conduction electrons can move only in order of their locations in the conduction channel, in case they are set to move. This orderly motion of electrons with their definite $\phi-$separation ($\phi = 2n\pi$) clearly represents a kind of coherence in their motion exhibited by their superconducting state. Further since $\phi = kr = 2n\pi$ implies that $k$ and $r$ of two electrons in superconducting state are inter-dependent, their binding in momentum space, obviously, represents a binding in real space. They have no kinetic energy above their zero-point energy of their localization in the channel since their $K-$motions get delinked from $q-$motions (*cf.* Section 7.2). Consequently, under the influence of their mutual repulsion and their interaction with other lattice constituents, their positions in the real space are expected to define a crystalline arrangement. Naturally, their mutual repulsion can also be an important factor to facilitate their motion in the order of their positions in a channel and this fact helps in identifying the basic difference of their superconducting state with their normal state where electrons are free to move randomly. One may identify this difference with the difference in the orderly movement of parading soldiers of an army platoon and the movement of people in a crowd.

## 6.11 Co-existence with other properties

Since the conduction electrons in their superconducting state get localized with an orderly arrangement in real space like atoms in a crystal (*cf.* Section 6.10) (*of course with a freedom to move in order of their positions*), they cease to have inter-electron collisions as well as collisions with lattice. Consequently, their spins can sustain their definite orientations leading to a magnetic state, such as diamagnetic or ferro-magnetic or anti-ferromagnetic state. The nature of the magnetic state is, obviously, decided by the different interactions with electrons spins and it can be understood by using the well known theories of diamagnetism, or ferro-magnetism or anti-ferromagnetism of a solid. Evidently, our theoretical framework finds no compelling reason for the superconducting state to be diamagnetic only. In fact the magnetic state of the superconducting electron fluid in a particular solid should be governed by the condition of minimum free energy with respect

to an appropriate order-parameter. Both, the diamagnetism of most superconductors and the co-existence of superconductivity with ferro-magnetism in fewer systems, could be a simple consequence of this condition. For the similar reasons, we may argue that pairing of electrons can also occur in triplet $p-$state or singlet $d-$state.

## 6.12 Principles of Superconductivity

Recently, Mourachkine [58] analyzed general principles of superconductivity from the standpoint of practical realization of RT superconductivity. He observes that : (i) RT superconductivity, if ever realized, would not be BCS type, (ii) the quasi-particle pairing which takes place in momentum space could possibly take place in real space and if it happens BCS theory and future theory of unconventional superconductors can hardly be unified, (iii) the mechanism of electron pair formation in all superconductors differs from the mechanism of Cooper pair condensation, (iv) the process of electron pairing precedes the process of Cooper pair condensation, *etc.* In this context our theoretical analysis reveals the following: (a) The main factor, which induces an indirect attraction between two conduction electrons necessary for the formation of Cooper type pairs, is a kind of mechanical strain in the lattice produced by the zero-point force of conduction electrons; while this fact supplements the BCS model in certain respect but at the same time it underlines the fact that the real mechanism of pairing of electrons responsible for superconductivity of widely different solids differs from BCS theory, (b) the quasi-particle electron pairing takes place not only in momentum space as envisaged by BCS model but in certain sense it, indirectly, occurs also in $r-$space (*cf.* Section 6.10) as well as in $\phi-$space, (c) While the conditions, in which electron pair formation is possible, exist at $T \leq T_o$ but the stabilization of such pairs (represented by an energy gap), which goes hand in hand with the onset of superconductivity or Cooper pair condensation occurs at $T_c$ (orders of magnitude lower than $T_o$); this clearly shows that the process of binding and the process of pair condensation are different and the former precedes the latter. As such these facts indicate that our inferences agree to a good extent with the basic principles of superconductivity as envisaged by Mourachkine [58]. However, in variance with his observation, BCS type model with mechanical strain produced by zero-point force as the main origin for the phonon induced interaction leading to Cooper type bound pairs of electrons unifies our understanding of the superconductivity of widely different superconductors.

### 7.0 Consistency With Other Important Theories

## 7.1 BCS theory

On the one hand our theoretical framework reinforces the basics of BCS picture (*viz.*, the formation of Cooper type pairs of electrons and their condensation as the origin of superconductivity), on the other hand it differs in certain respect. For example, it identifies the mechanical strain in the lattice produced by the zero-point force of conduction electrons (Sections 3.4.5 and 6.2) as the main factor responsible for the phonon induced attraction between two electrons. It is evident that the electrical strain emphasized by BCS model too contributes to this attraction but it may noted that the mechanical strain alone predicts a $T_c \approx 124K$ (*cf.* Section 6.6), while the electrical strain in BCS picture

accounts for a $T_c < \approx 25$K only. Assuming that both strains contribute in all systems and these two values represent their proportional contributions, one finds that electrical strain contributes only around 18%. Further it is important to note that the lattice in the superconducting phase stores an additional potential energy as the strain energy, $E_s$ (*cf.* Section 6.8), when the energy of net system (conduction electrons + lattice) falls with the onset of Cooper type pair formation. Since $E_s$ stays with the lattice even at $T = 0$ at which no phonon exists in the system, this energy becomes a source for the creation of phonons necessary to mediate correlated motion of two electrons of a Cooper type pair at all $T \leq T_c$ including $T = 0$. As such this section not only identifies the basic differences of our theory with BCS theory but also underlines the fact that our theory incorporates BCS theory. Different aspects of superconducting phase such as coherence length, ctrical current, critical magnetic field, persistence of current, *etc.* which depend on $E_g(T)$ can, therefore, be understood by using the appropriate relations available from BCS theory.

## 7.2 Two fluid theory

We note that: (i) each electron represented by a macro-orbital has two motions, $q$ and $K$, (ii) they have separate free energy contributions, $N\varepsilon_o$ and $A'$ and (iii) the onset of superconductivity locks the $q-$motions of all electrons at $q = q_o$ with an energy gap which isolate them from $K-$motions. Evidently, the superconducting state of the fluid at $T \leq T_c$ can be described by

$$\Psi^S(N) = \Pi_i^N \zeta_{q_o}(r_i) \sum_P^{N!} (\pm 1)^P \Pi_i^N \exp\left[i(P\mathbf{K}_i\mathbf{R}_i)\right] \tag{47}$$

which has been obtained by using all $q_i = q_o$ in Eqn. 28. We note that $\Psi^S(N)$ is product of two separate functions, *i.e.*, $\Psi^S(N) = \psi_K(N)\psi_q(N)$ with

$$\psi_K(N) = \sum_P^{N!} (\pm 1)^P \Pi_i^N \exp\left[i(P\mathbf{K}_i\mathbf{R}_i)\right] \tag{48}$$

and

$$\psi_q(N) = \Pi_i^N \zeta_{q_o}(r_i) \tag{49}$$

This implies that the electron fluid at $T \leq T_c$ can be identified as a homogeneous mixture of two fluids: (F1) described by $\psi_K(N)$ where electrons represent some sort of quasi-particles described by plane waves of momentum $K$ and (F2) described by $\psi_q(N)$ where each electron represents a kind of localized particle in ($\mathbf{q}$, -$\mathbf{q}$) bound pair state where it ceases to have collisional motion. With all electrons having $q = q_o$, F2 represents the $q-$motion ground state where each electron has no thermal energy (*i.e.*, no energy above the zero-point energy, $\varepsilon_o$) and it has *zero entropy* which is also supported by the fact that the number of different configurations with all particles having $q = q_o$ is only 1. We further note that particles in F2 are basically localized; if they are set to move they move in order of their location with no relative motion, no collision or scattering. Naturally, they find no reason (such as collision or scattering) to lose their energy indicating that their flow should be resistance free implying that F2 is in superconducting phase. Since each particle in this phase has an energy gap ($\epsilon_g$) with respect to its state in normal phase at $T_c^+$ (just above $T_c$), the former is stable against any perturbation of energy $< \epsilon_g$.

Naturally, when this fact is clubbed with the coherent motion of macroscopically large number of electrons it becomes evident that the source resistance should be strong enough to reduce the velocity all such electrons in a single event which however is not possible. As such we find that F1 and F2 at all $T \leq T_c$ have all properties that have been envisaged by Landau [59] in the normal fluid and superfluid components of an electron fluid which implies that our theory provides microscopic foundations for the two fluid phenomenology. Since Bardeen [60] has elegantly analyzed BCS theory as the microscopic basis of two fluid theory and our theory incorporates BCS model, his results can be used to determine the normal ($\rho_n$) and superfluid density ($\rho_s$) components needed for implementation of two fluid theory.

### 7.3 $\Psi-$ Theory

We note that superconductivity is basically a property of F2 in its ground state which represents the relative configuration of electrons in ($\mathbf{q}$ -$\mathbf{q}$) pair states with $q = q_o$ which is, evidently, described by

$$\Psi_o(N) = \Pi_i^N \zeta_{q_o}(r_i) = \sqrt{\mathrm{n}} \tag{50}$$

(with n $= N/V$ being the electron number density). To obtain Eqn. 50 we separated the $K-$dependent part of $\Psi^S(N)$ (Eqn. 47) describing F1. Since each electron in ($\mathbf{q}-\mathbf{q}$) configuration under the influence of any perturbation that makes it move with a momentum say $\Delta \mathbf{K}$ assumes ($\mathbf{q} + \Delta \mathbf{K}$, -$\mathbf{q} + \Delta \mathbf{K}$) configuration and its state is described by

$$\zeta(r, R) = \zeta_{q_o}(r) \exp\left(i\mathbf{Q}.\mathbf{R}\right) \tag{51}$$

with $\mathbf{Q} = 2\Delta\mathbf{K}$, it is evident that superconducting state under such perturbation would be described by

$$\Psi'_o(N) = \Pi_i^N \zeta_{q_o}(r_i) \exp\left(i\Phi\right) = \sqrt{\mathrm{n}} \exp\left(i\Phi\right) \tag{52}$$

with its phase $\Phi = \sum_i^N \mathbf{Q}_i.\mathbf{R}_i$ and $\mathbf{Q}_i = 2\Delta\mathbf{K}_i$. However, for the phenomenological reasons (*viz.* the number density of superconducting electrons ($\mathrm{n}_s$) need not be equal to n) we replace $\Phi$ by $\Phi + i\Phi'$ and recast $\Psi'_o(N)$ as

$$\Psi'_o(N) = \sqrt{n_s} \exp\left(i\Phi\right) \tag{53}$$

which renders $\mathrm{n}_s = \mathrm{n} \exp\left(-2\Phi'\right)$. We note $\Psi'_o(N)$ clearly has the structure of $\Psi-$function that forms the basis of the well known $\Psi-$theory of superfluidity. This shows that our theory provides microscopic foundation to the highly successful $\Psi-$theory [20].

### 7.4 Theory based on the proximity of a QPT

In view of Sections 7.2 and 7.3, superconductivity is a property of F2 system in its ground state obviously representing a state at $T = 0$K. This implies that superconducting transition is, basically, a *quantum phase transition* that occurs in F2 system exactly at $T = 0$ but its proximity with F1 makes it appear at non-zero $T$ in the real system (a homogeneous mixture of F1 and F2 where each particle participates identically); they manifest as two separated fluids at all $T \leq T_c$ for the presence of the energy gap, while such separation ceases to exist at $T > T_c$. Evidently, our theory is consistent with the idea

which relates superconductivity with the proximity effect of a quantum phase transition [15].

## 8.0 Concluding Remarks

This paper uses a new approach to lay the basic foundations of superconductivity. It finds that each conduction electron (particularly in low energy states) is more accurately represented by a macro-orbital (*cf.*, Section 3.4.7) rather than a plane wave. While it reinforces the basics of the BCS model of superconductivity (*viz.*, the formation of $(\mathbf{q}, -\mathbf{q})$ bound pairs of conduction electrons and their condensation) but it also reveals that the basic reason for the phonon induced attraction between two electrons (responsible for the formation of Cooper type bound pairs) rests with a mechanical strain in the lattice produced by zero-point force (a well known force of a spatially confined quantum particle in its ground state). The electrical strain in the lattice produced by the electric charge of electrons can have its contribution to such attraction and resulting binding of two electrons as Cooper type pair. In principle, other interactions such as spin-spin, spin-latice, *etc.* too can add to the said binding which, obviously, implies that our approach accomodates all possible interactions that may contribute to bound pair formation.

In agreement with the BCS model, our approach also finds an energy gap (between superconducting and normal states of the electron fluid) resulting from the phonon induced attraction between two electrons in $(\mathbf{q}, -\mathbf{q})$ configuration. Evidently, our framework can identically account for all aspects of superconducting phase that are accounted for by BCS theory and for this reason we need not rederive the relations which connect $E_g(T)$ with various properties of a superconductor and restate the scientific arguments which help in their physical understanding. It is, however, important to note that our theory concludes a simple relation for $T_c$ (*cf.*, Eqn. 46) which not only accounts for the highest $T_c$ that we know to-day but also reveals a possibility of observing superconductivity at RT.

The diameter/ width of the narrow channels ($d_c$) through which conduction electrons flow in a solid plays an important role in controlling superconducting $T_c$ (Eqn. 46), while the process through which conduction electrons come into existence at a $T \geq T_c$ is unimportant. Our approach is also applicable to the systems with holes as the charge carriers because the flow of holes is nothing but the flow of electrons (*once again through the narrow channels*) by way of hopping between successive electron vacancies.

We find that superconductivity is basically a property of the ground state of electrons as $(\mathbf{q}, -\mathbf{q})$ bound pairs with $q = q'_o = \pi/d'_c$. The system specific or class specific properties can be obtained by using appropriate term(s) from $V'(N)$ as perturbation on the states of $H_o(N)$ (Eqns. 1 and 2). As such our approach does not forbid: (i) pair formation in triplet $p-$state and singlet $d-$state as well as (ii) the coexistence of superconductivity with ferro-magnetism or anti-ferromagnetism.

Although, this paper does not analyze the origin of experimentally observed pseudo-gap, charge stripes, *etc.* in HTC systems, however, it appears that these aspects do not have *direct* relation with the origin of superconductivity. We would like to examine the physics behind these observations as part of our future course of studies.

Finally, it may be mentioned that our theory only assumes that, to a good approximation, conduction electrons can be identified as HC particles of a Fermi fluid which flows through narrow channels (cylindrical tubes or 2-D slots in the lattice structure). It makes no presumption about the nature of the microscopic mechanism of superconductivity. Its all inferences are drawn from a systematic analysis of the solutions of the Schrödinger equation of $H_o(N)$. The fact, that a system described by $H_o(N)$ exhibits superfluidity if its particles have inherent or induced inter-particle attraction and the system remain fluid at $T \leq T_c$, has been successfully demonstrated in [42] by using the same approach. The mathematical foundation and formulation of our theoretical framework are simple and it has great potential for developing equally simple understanding of different aspects of superconductivity and related behavior of widely different superconductors.

Over the last two decades, one of the major thrusts of researches in the field of superconductivity has been to find the basic mechanism which can account for the experimentally observed high $T_c$. One may find that the prersent work has been able to achieve this objective. Since it provides clear picture of the ground state configuration of electrons, it may help in studying the details of inter-particle correlations at $T \leq T_c$ in $q-$, $\phi-$ and $r-$spaces required for understanding their transport properties in superconducting phase; however, such studies could be taken only in a future course of our research. As evident from Sections 5.0 and 6.0, the present study also reveals that electron fluid in solids should behave almost like: (i) a system of non-interacting fermions at $T > T_a$ when electrons can be represented by plane waves, (ii) a Landau-Fermi liquid (with quasi-particle mass $= 4m$ which may, however, be modified due to interacting environment of the electrons) at $T_a > T > T_c$ when they are better represented by macro-orbitals, and (iii) a singular Fermi liquid at $T \leq T_c$ when $N\varepsilon_o$ becomes critical under the influence of zero-point force. Varma *et.al.* [56] have elegantly itroduced the subject related to these three phases of the behavior of a system of interacting fermions.

The present theory is, obviously, applicable to any other system of HC fermions with weak inter-particle attraction (*viz.* liquid $^3He$) by simply assigning the role of *lattice structure* to the atomic arrangement of neighboring $^3He$ atoms around a chosen $^3He$ atom whose $q_o$ is now decided by $d = (V/N)^{1/3}$. In this context, it may be mentioned that no other theory has been able to obtain superfluid $T_c$ for liquid $^3He$ which falls so close to its experimental value as found by us (*cf.* Section 6.6). Although our account of this system is not expected to differ from that found by using BCS model because our theory incorporates BCS model but for the first time it identifies zero-point force leading to a strain in $He - He$ bonds as the origin of Cooper type bound pairs of $^3He$ atoms responsible for the superfluidity of liquid $^3He$. While the salient aspects of our study of liquid $^3He$ (presented in a recent conference) are available in [42], the detailed would soon be published elsewhere.

*Note :* The author would greatly appreciate the comments of the scientific community on this simple approach to the understanding of superconductivity of widely different systems.

## Appendix - A

## A Critical Analysis of $< A\delta(x) >$

The relative motion of two particles interacting through a central force potential basically represents a 1-D motion along the line joining their centers of mass. The following analysis uses this observation to establish the validity of $< V_{HC}(r) >=< A\delta(r) >= 0$ (Eqn. 18) for all possible physical situations in relation to the relative motion of two HC particles in 1-, 2- and 3-dimensions by considering the 1-D analogue ($< V_{HC}(x) >=< A\delta(x) >=$) of Eqn. 18. We note $A$ in $V_{HC}(x) \equiv A\delta(x)$ is such that $A \to \infty$ for $x \to 0$ and it can in general be expressed as

$$A = Bx^{-(1+\alpha)} \qquad (A-1)$$

where both $B$ and $\alpha$ are $> 0$. Using the pair state $\zeta^-$ or $\zeta^+$ (Eqns. 13 and 14), we find that

$$< A\delta(x) >= B\frac{2\sin^2(kx/2)}{x^{(1+\alpha)}}\Big|_{x=0} \qquad (A-2)$$

is an in-determinant which can be simplified to $Bk^2 x^{1-\alpha}/2$ for $x \approx 0$. Evidently, when $x \to 0$, $< A\delta(x) >$ has 0 value for $\alpha < 1$, a +ve value ($= Bk^2/2$) for $\alpha = 1$ and $\infty$ for $\alpha > 1$. Since no physical system can ever occupy a state of $\infty$ potential energy, $\alpha > 1$ corresponds to a physically uninteresting case. While remaining $\alpha$ values correspond to physically possible configurations, $\alpha = 1$ is the sole point on the $\alpha-$line for which $< A\delta(x) >$ assumes a finite +ve value. In fact $\alpha = 1$ stands as a sharp divide between the states of $< A\delta(x) >= 0$ and $< A\delta(x) >= \infty$. To understand the physical significance of these results, we note the following.

1. $< A\delta(x) >= 0$ for $\alpha < 1$ implies that Eqn. 18 is clearly valid for this range of $\alpha$.

2. $< A\delta(x) >= Bk^2/2$ for $\alpha = 1$ renders

$$E^* = \frac{\hbar^2 k^2}{4m} + \frac{Bk^2}{2} = \frac{\hbar^2 k^2}{4m}\left(1 + \frac{2Bm}{\hbar^2}\right) \qquad (A-3)$$

which, *in principle*, represents the total energy expectation of the relative motion of two HC particles interacting through $A\delta(x)$. One may write $E^* = \hbar^2 k^2/4m^*$ to absorb $< A\delta(x) >= Bk^2/2$ and $\hbar^2 k^2/4m$ into a single term by defining $m^*$ as

$$m^* = \frac{m}{1 + 2Bm/\hbar^2} \qquad (A-4)$$

and use $< A\delta(x) >= 0$. While this shows that our results, interpretations and conclusions based on Eqn. 18 are valid even for $\alpha = 1$ if $m$ is replaced by $m^*$, however, it does not explain why $E^*$ far from $x = 0$ should be different from $E_k = \hbar^2 k^2/4m$ and why $< A\delta(x) >$ (as indicated by its proportionality to $k^2$) should be kinetic in nature; it may be noted that $< A\delta(x) >= Bk^2/2$ does not have potential energy character of $A\delta(x)$ because it is neither a function of $x$ nor of $< x >$. Evidently, $< A\delta(x) >= Bk^2/2$ needs an alternative explanation (*cf.* points 3-5 below).

3. Two particles in their relative motion have only kinetic energy $(E_k = \hbar^2 k^2/4m)$ till they reach the point of their collision at $x = 0$ where they come to a halt and $\hbar^2 k^2/4m$ gets transformed into an equal amount of potential energy (as a result of energy conservation), naturally, proportional to $k^2$ as really found with $< A\delta(x) >= Bk^2/2$. This mplies that $< A\delta(x) >= Bk^2/2$ does not represent an additional energy to be added to $- < (\hbar^2/m)\partial_x^2 >= \hbar^2 k^2/4m$ in determining $E^*$ as found in Eqn. A-3. To this effect we find that the physical meaning of non-zero $< A\delta(x) >$ of an ill behaved potential function $A\delta(x)$ may differ from that of $< V(x) >$ of a well behaved (*i.e.* continuous and differentiable) potential function, $V(x)$.

4. We also find that $< A\delta(x) >= Bk^2/2$ is independent of the limits of integration $x^-$ and $x^+$ (with $x = 0$ falling between $x^-$ and $x^+$), even when we use $x^- = -\epsilon$ and $x^+ = +\epsilon$ with $\epsilon$ being infinitely small. In other words $< A\delta(x) >$ has solitary contribution $(=Bk^2/2)$ from $x = 0$, while $- < (\hbar^2/m)\partial_x^2 >= \hbar^2 k^2/4m$ (kinetic energy) has zero contribution from this point; in fact $- < (\hbar^2/m)\partial_x^2 >= \hbar^2 k^2/4m$ is independent of the inclusion or exclusion of $x = 0$ in the related integral. Evidently, the energy measured as $- < (\hbar^2/m)\partial_x^2 >$ appears as non-zero $< A\delta(x) >$ at $x = 0$ and $E^*$ should be simply equal to $- < (\hbar^2/m)\partial_x^2 >$ by treating non-zero $< A\delta(x) >$ as ficitious that could be assumed to be zero for all practical purposes; this falls in line with an important observation by Huang [47] that HC potential is no more than a boundary condition for the relative wave function.

5. In the wave mechanical framework, two colliding particles either exchange their positions (across the point $x = 0$) or their momenta. In the former case they can be seen to cross through their $\delta-$potential possibly by some kind of tunneling (in which their kinetic energy does not transform into potential energy), while in the latter case they return back on their path after a halt at $x = 0$ in which case their potential energy rises at the cost of their kinetic energy. It appears that the two possibilities can be, respectively, identified with $< A\delta(x) >= 0$ and $< A\delta(x) >= Bk^2/2$, however, one has no means to decide whether the two particles exchanged their positions or their momenta which implies that the two situations are indistinguishable and $< A\delta(x) >$ can be measured to have 0 to $Bk^2/2$ values. Apparently this is not surprising since the state of a collision of two HC particles at $x = 0$ (*i.e.* an exact $x$) is a state of zero uncertainty in $x$ and infinitely high uncertainty in momentum $k$ or in energy $E_k = \hbar^2 k^2/4m$.

In summary non-zero $< A\delta(x) >= Bk^2/2$ observed for $\alpha = 1$ should treated as fictitious. It can best be attributed to energy conservation at $x = 0$. This implies that $< A\delta(x) >= 0$ (*i.e.* Eqn. 18) is relevant both for $\alpha < 1$ and $\alpha = 1$.

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Consider a system of :(i) a quantum particle of mass $m$ and (ii) a 1-D oscillator (a particle of mass $M$ attached to a spring of length $a$ and force constant $C$) placed side by side in a 1-D box of length $l$ with both being in their ground states; they do not share any coordinate $x$ in the box with the former representing a particle trapped in a 1-D box of size $d = l - a$. Evidently, we have $E_o = h^2/8md^2 + 0.5\hbar\omega$ (with $\omega = \sqrt{C/M}$) as ground state energy of the system. However, under the zero point force of the particle, the length of the oscillator is contracted by $\Delta d = d' - d$ and the effective size of the box for quantum particle becomes $d'$. Consequently, to a good approximation, $E_o$ changes to $E_o' = h^2/8md'^2 + 0.5\hbar\omega + 0.5C\Delta d^2$ and use of the equilibrium condition $f_o = h^2/4md'^3 = C\Delta d$, renders: (i) $\Delta E = E_o' - E_o = -(h^2/8md^3)(\Delta d/d)$ as the net decrease in the energy of the system, (ii) $\Delta\varepsilon_o = $

$-(h^2/4md^3)(\Delta d)$ as the net decrease in the zero-point energy of the quantum particle and (iii) $0.5C\Delta d^2 = +(h^2/8md^3)(\Delta d)$ as the net energy added to the spring as its strain energy.

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